Exercise 7.1

The heat flux can be easily calculated with the given heating power (3 kW), efficiency (60%), and bottom surface area (30 cm diameter) of the pan,

$$q = 3 \times 10^3 \;\times\; 60\% \;\div \left(\frac{\pi \times 0.3^2}{4}\right) = 25478\; W \cdot m^{-2}$$

As the pressure is 1 atm, the boiling point water $T_{sat} = 100°\mathrm{C}$. Then, the inner surface temperature can be estimated by the figure in Slide 6 or calculated using Equation 7.1 knowing that $\Delta T_{excess} = T_s - T_{sat}$.

In the figure in Slide 6, with a boiling heat flux $q = \sim 2.5 \times 10^4\; W/m^2$, $\Delta T_{excess} = 7°\mathrm{C}$ can be roughly estimated. Thus, the inner surface temperature of the pan $T_s = 107°\mathrm{C}$.

Alternatively, with equation 7.1, at $100°\mathrm{C}$, the surface tension of liquid-vapor interface $\sigma = 0.0589 N/m$, and for mechanically polished stainless steel, $C_{sf} = 0.0130$ and $n = 1.0$. Together with all give physical properties of water, we have

$$25477 = 0.282 \times 10^{-3} \times 2257 \times 10^3 \times \left[\frac{9.81 \times (957.9 - 0.6)}{0.0589}\right]^{1/2} \times \left(\frac{4217 \times \Delta T_{excess}}{0.013 \times 2257 \times 10^3 \times 1.75}\right)$$

$$\Delta T_{excess} = 5.7°\mathrm{C}$$

Thus, the inner surface temperature of the pan $T_s = 105.7°\mathrm{C}$.

To estimate the temperature difference between the inner and outer surface of the pan, we consider 1-D Fourier's law,

$$q = -k\frac{dT}{dx}$$

With constant k, simple integration gives

$$q(x_2 - x_1) = -k\Delta T$$

Where $x_2 - x_1 = 6\; mm$ (thickness of the pan). Thus,

$$\Delta T = \frac{25478 \times 6 \times 10^{-3}}{16.2} = 9.4°\mathrm{C}$$

For effective h between the pan and the water, we know that

$$q = h(T_s - T_{sat})$$

Thus,

$$h = \frac{25478}{5.7} = 4470\ W \cdot m^{-2} \cdot K^{-1}$$

Exercise 7.2

Given that

$$L_c = L + \frac{t}{2} = 0.04 + \frac{0.002}{2} = 0.041m$$

$$\frac{L_c + r_1}{r_1} = \frac{0.041 + 0.04}{0.04} = 2.025$$

$$L_c(\frac{h}{kt})^{1/2} = 0.041 \times \sqrt{\frac{30}{200 \times 0.002}} = 0.355$$

In the graph, we can find that $\eta_f = 0.88$

The surface area of the fin is

$$A = 2[\pi(L + r_1)^2 - \pi r_1^2] + 2\pi(L + r_1)t = 0.03115m^2$$

Thus, the heat loss is

$$Q_{loss} = hA(T_s - T_\infty)\eta_f = 30 \times 0.03115 \times (523.2 - 343.2) \times 0.88 = 148W$$